Printed Page:- Subject Code:- BAS0104

Roll. No:

NOIDA INSTITUTE OF ENGINEERING AND TECHNOLOGY, GREATER NOIDA

(An Autonomous Institute Affiliated to AKTU, Lucknow)

B.Tech

SEM: I - THEORY EXAMINATION (2023-2024)

Subject: Mathematical Foundations-I

Time: 3 Hours Max. Marks: 100

General Instructions:

IMP: *Verify that you have received the question paper with the correct course, code, branch etc.*

1. This Question paper comprises of three Sections -A, B, & C. It consists of Multiple Choice Questions (MCQ's) & Subjective type questions.

2. Maximum marks for each question are indicated on right -hand side of each question.

3. Illustrate your answers with neat sketches wherever necessary.

4. Assume suitable data if necessary.

5. Preferably, write the answers in sequential order.

6. No sheet should be left blank. Any written material after a blank sheet will not be evaluated/checked.

SECTION-A 20

1. Attempt all parts:-

1-a. If a matrix of order *m x n* can be expressed as $A = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$, the rank of A is (CO1) 1

(a) r

(b) r+1

(c) r-1

(d) none of these

1-b. If the eigen values of a matrix A are 4, 5, 7 then write the eigen values of A^{-1} are (CO1) 1

(a) $4, 5^2, 7^3$

(b) 4, 5, 7

(c) 1/4, 1/5, 1/7

(d) none of these

1-c. The dimension of zero vector space is _. (CO2) 1

(a) Not defined

(b) 1

(c) 0

(d) infinite

1-d. If V be an inner product space, then distance between two vector α and β is _. (CO2) 1

(a) $\|\alpha-\beta\|$

(b) $\|\alpha+\beta\|$

(c) $\|\alpha\|\|\beta\|$

(d) None of these

1-e. If $y=\cos^{-1}x$ then $y_2(1-x^2)-xy_1$: (CO3) 1

(a) 0

(b) 1

(c) 2

(d) 3

1-f. If $f(x,y,z)=x^2+xyz+z$, then $\frac{\partial f}{\partial x}$ at $(1,1,1)$ (CO3) 1

(a) 0

(b) 3

(c) 1

(d) -1

1-g. Maclaurin's series for f(x) is (CO4) 1

(a) $f(0)+\frac{x}{1!}f'(0)+\frac{x^2}{2!}f''(0)+\frac{x^3}{3!}f'''(0)+.....$

(b) $f(x)+\frac{x}{1!}f'(x)+\frac{x^2}{2!}f''(x)+\frac{x^3}{3!}f'''(x)+.....$

(c) $f(0)+\frac{x}{1}f'(0)+\frac{x^2}{2}f''(0)+\frac{x^3}{3}f'''(0)+.....$

(d) $f(x)+\frac{1}{1!}f'(x)+\frac{1}{2!}f''(x)+\frac{1}{3!}f'''(x)+.....$

1-h. Stationary poins of the function $f(x,y)=x^2+y^2+6x+12$ is (CO4) 1

(a) (0, 0)

(b) (6, 0)

(c) (3, 0)

(d) (-3, 0)

1-i. 600 is increased by 20% then the new number is (CO5) 1

(a) 720

(b) 700

(c) 750

(d) None of these

1-j. Average of first 10 natural numbers = (CO5) 1

(a) 5

(b) 6

(c) 5.5

(d) 5.2

2. Attempt all parts:-

2.a. Show that the matrix $\begin{bmatrix} 3 & 7-4i & -2+5i \\ 7+4i & -2 & 3+i \\ -2-5i & 3-i & 4 \end{bmatrix}$ is Hermitian matrix. (CO1) 2

2.b. Define the term Nullity for a linear transformation. (CO2) 2

2.c. Find the n th differential coefficients of $x^2 \sin 3x$ (CO3) 2

2.d. Expand sin x in powers of $\left(x - \frac{\pi}{2}\right)$. (CO4) 2

2.e. John made a profit of 25% while selling a book for Rs.250. Find the cost price of the book? (CO5) 2

SECTION-B 30

3. Answer any five of the following:-

3-a. Find the eigen values and eigen vectors of a matrix $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$. (CO1) 6

3-b. Find the rank of matrix by reducing it to normal form $\begin{bmatrix} 1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 1 & 2 & -8 \end{bmatrix}$. (CO1) 6

3-c. Show that the mapping $T: V_3(R) \to V_2(R)$ *defined as* $T(a, b, c) = (a, b)$ is a linear transformation. (CO2) 6

3-d. Is the vector (2, -5, 3) in the subspace of R^3 spanned by the vectors (1, -3, 2), (2, -4, -1), (1, -5, 7). (CO2) 6

3.e. If $\log u = \frac{x^3+y^3}{3x+4y}$ then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2u \log u$ (CO3) 6

3.f. Expand $f(x, y) = x^2 + 3y^2 - 9x - 9y + 26$ as Taylor's series expansion about the point (1,2) upto three terms. (CO4) 6

3.g. Water tax is increased by 20% but its consumption is decreased by 20%.Then, find the increase or decrease in the expenditure of the money ? (CO5) 6

SECTION-C 50

4. Answer any one of the following:-

4-a. Investigate for what values of λ and μ the equations $x + 2y + z = 8$, $2x + 2y + 2z = 13$ and $3x + 4y + \lambda z = \mu$ have (i) no solution (ii) unique solution (iii) many solutions. (CO1) 10

4-b. Show that the matrix $A = \begin{bmatrix} 1 & -2 & 1 \\ 1 & -2 & 3 \\ 0 & -1 & 2 \end{bmatrix}$ satisfies its own characteristic equation and hence find A^{-1}. (CO1) 10

5. Answer any one of the following:-

5-a. Find the range, rank, kernel and nullity of the linear transformation $T : R^3 \rightarrow R^2$ defined by $T(a_1, a_2, a_3) = (a_1+a_2, 2a_3 - a_1)$. (CO2) 10

5-b. Show that the vectors (1, 1, 2, 4), (2, -1, -5, 2), (1, -1, -4, 0) and (2, 1, 1, 6) are linearly independent in R^4 . (CO2) 10

6. Answer any one of the following:-

6-a. If $y = e^{a \cos^{-1}x}$, then prove that 10
$(1 - x^2) y_{n+2} - (2n+1) x y_{n+1} - (n^2 + a^2) y_n = 0.$
Calculate $y_n(0)$ (CO3)

6-b. If $u = f(2x - 3y, 3y - 4z, 4z - 2x)$, then prove that $\frac{1}{2}\frac{\partial u}{\partial x} + \frac{1}{3}\frac{\partial u}{\partial y} + \frac{1}{4}\frac{\partial u}{\partial z} = 0$ 10
(CO3)

7. Answer any one of the following:-

7-a. If $u = \frac{x+y}{z}, v = \frac{y+z}{x}, w = \frac{y(x+y+z)}{xz}$, then show that u, v, w are not independent and find the relation between them. (CO4) 10

7-b. Find the maximum and minimum distances from the point (1, 2, -1) to the sphere $x^2+y^2+z^2 = 24$. (CO4) 10

8. Answer any one of the following:-

8-a. (a) If the sum of first p terms of an arithmetic progression is equal to sum of first q terms of the same arithmetic progression, then what is the sum of (p + q) terms? Given that p is not equal to q. 10
(b) In a certain code, HARYANA is written as 8197151, how is DELHI written in that code? (CO5)
(c) In copying a quadratic equation of the form $x^2 + px + q = 0$, the coefficient of x was wrongly written as -10 instead of -11 and the roots were found to be 4 and 6. Find the roots of the correct equation ? (CO5)

8-b. .(a) A student who secures 20% marks in an examination fails by 30 marks. Another student who secures 32% marks gets 42 marks more than those required to pass. Find the percentage of marks required for pass ? 10
(b) The average of marks obtained by 120 candidates was 35. If the average of marks of passed candidates was 39 and that of failed candidates, was 15 .Find the number of candidates who passed the examination?
(c) Find the missing term of the given series 15, 32, 93, 460, ?, 35354 (CO5)